Low Complexity Pilot Decontamination via Blind Signal Subspace Estimation

L. Cottatellucci
laura.cottatellucci@eurecom.fr

joint work with R. Müller, and M. Vehkaperä
I. Outline

Outline

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II. Motivations

MIMO Cellular Systems

Cooperative approach:

- Space division multiple access inside a cell
- Channel sharing among cells is spectral efficient but...
- ...interference management highly costly

\[
\begin{align*}
\text{Data sharing;} \\
\text{Channel state information acquisition;} \\
\text{Signalling.}
\end{align*}
\]
II. Motivations

A General System Model

\[ y(m) = H x(m) + n(m) \]

- Multiuser CDMA;
- Multiuser SIMO;
- Single/Multiuser MIMO.
• At very low loads all detectors have equal performance.
• Matched filter: only knowledge of channel for user of interest needed.
• MMSE detector: statistical knowledge of all channel required.

At very low load matched filter optimally combats interference without coordination/cooperation.
II. Motivations

Massive MIMO Concept

- Huge antenna arrays \( (R \gg 1 \text{ antennas}) \) at the base stations serving a few users \( (T \ll R \text{ users}) \)
- Under assumption of perfect channel knowledge and \( T/R \to 0 \), beams can be made sharper and sharper and interference vanishes.

**Interference management without coordination or cooperation!**
II. Motivations

Pilot Contamination for TDD Systems

Simple scenario

- Users send orthogonal pilots within a cell, but the same training sequences are used in adjacent cells.
- By channel reciprocity, the channel estimates are useful for both uplink detection and downlink precoding.
II. Motivations

Pilot Contamination

Simple channel estimation (Marzetta '10)

- Linear channel estimation by decorrelator/matched filter is limited by copilot interference.
- Subsequent detection or precoding based on the low quality channel estimates degrade significantly the system spectral efficiency.
Proposed Countermeasures: State of Art

- Coordinated scheduling among cells.
- Coordinated training sequence assignment.

...but coordination very costly and complex in terms of signaling!
II. Motivations

A Deeper Look at the Impairment

• In the simple Marzetta’s scheme, array gain is utilized for data detection but not for channel estimation.

• Linear channel estimation does not exploit the array gain.

Guidelines for Countermeasures

• General channel estimation that utilizes the array gain.
System Model I

- $L$ interfering cells
- $T$ transmitters
- $R$ receive antennas

$R \gg T$
$R \gg T(L+1)$
III. System Model

System Model II

Assumptions

- **Power control** such that in-cell users’ signals are received with equal power \( P \).
- **Handover** to guarantee that \( P > I \).
System Model for Channel Estimation

\[ Y = HX + W \]

- \( C \): coherence time.
- \( Y \): \( R \times C \) matrix of received signals.
IV. Subspace Approach

Projection Subspace

\( \mathbf{Y} \mathbf{Y}^H \) is a matrix with \( T \) positive eigenvalues and \( R - T \) zero eigenvalues.

Let \( S \) be the \( R \times T \) matrix of eigenvectors corresponding to the nonzero eigenvalues:

- \( S \) spans the signal subspace;
- \( \mathbf{Y}' = \mathbf{S}^H \mathbf{Y} \) is the projection of the received signal into the signal space;
- We can estimate the equivalent channel in the \( T \) dimensional signal subspace \( S \) using \( \mathbf{Y}' \) without performance loss.
IV. Subspace Approach

**Projection Subspace**

In the presence of additive Gaussian noise and \( C \) sufficiently large

- The matrix \( S \) consisting of the \( YY^H \) eigenvectors corresponding to the \( T \) largest eigenvalues is still a basis of the signal subspace;

- By using the projection \( Y' = S^H Y \), the white noise impairing the observed signal is reduced from \( R\sigma^2 \) to \( T\sigma^2 \)

- In massive MIMO, since \( R \gg T \) and \( T/R \rightarrow 0 \) the noise is negligible compared to the signal power.
  - \( S \) spans the signal subspace;
  - \( Y' = S^H Y \) is the projection of the received signal into the signal space;
  - We can estimate the equivalent channel in the \( T \) dimensional signal subspace \( S \) using \( Y' \) without performance loss.

**Fully blind method to obtain array gain!**
Projection Subspace Method

✓ In the presence of additive Gaussian noise and intercell interference

− If $T/R \to 0$ and $P > I_k$ the signals of interest and the interferences are almost orthogonal.
− There will be two disjoint clusters of eigenvalues with the $T$ highest eigenvalues associated to the signal of interest.

✓ The same projection method can be applied also in this case.

✓ Interference power subspace and white noise become negligible!

Pilot contamination is not a fundamental issue in massive MIMO!
How this method can be extended to practical systems with a finite number of receive antennas and finite coherence time?
Eigenvalue Spectrum of $YY^H$ for Practical Systems

If the eigenvalue spectrum of $YY^H$ consists of disjoint bulks associated to the interference and desired signals, the subspace method can still be applied and suppresses the most of interference and noise also when $T/R = \alpha > 0$ and $C/R = \kappa < +\infty$.

Fundamental to study the eigenvalue spectrum!

We approximate a system with finite $T, R, C$ by a system with $T, R, C \rightarrow +\infty$ and $T/R \rightarrow \alpha$ and $R/C \rightarrow \kappa$. 
**Eigenvalue Distribution of Observation Signal Covariance**

Solid red line: Asymptotic eigenvalue distribution by random matrix theory

\[ \alpha = 1/100, \ \kappa = 10/3, \ r = 1/100, \ t = 4/100, \ T = 3, \ R = 300, \ C = 1000, \ P = 0.1, \ I = 0.025, \ W = 1 \]
V. Subspace Method in Practical Systems

Analysis of the Eigenvalue Bulk Gap

✓ Assume worst case with interferers received at the maximum power $I < P$.
✓ Let $\beta = I/P$.
✓ Approximate the eigenvalue distribution finite systems by asymptotic eigenvalue distribution.

**Conservative** condition for a nonzero gap btw interference and signal bulks

$$\frac{T}{C} \leq \frac{(1 - \beta)^2(L\beta^2 + 3(L + 1)\beta + 1 - 2(1 + \beta)\sqrt{3L\beta})}{(L\beta^2 - 1)(L\beta^2 + 6(L - 1)\beta - 1) + (9L^2 - 2L + 9)\beta^2}.$$ 

✓ Dependent only on the ratio $T/C$!
✓ Independent of $R$!
Separability Region

Region of separability for signal and interference subspaces

- L=2
- L=4
- L=7
Coherence Time vs Receive Antenna

- $T = 5,$
- $C = 100,$
- $L = 2,$
- $\frac{P}{W} = 0.1$ (SNR $= -10$ dB)
- $I_{\text{max}}/P = 0.5$

- $C$ not required to scale with $R$ for bulk separability.
- For a given $C$, an increase in $R$ helps.
VI. Performance Assessment

**Projection Subspace Method vs Linear Estimation**

- $T = 5,$
- $C = 100,$
- $L = 6,$
- $P/W = 0.1$ (SNR $= -10$ dB)
- $I_k = \frac{kP}{\delta T}$
- $I_{\text{max}} = \frac{P}{\delta}$
- $\delta = 2, \ldots, 6$

Subspace projection method benefits from an increase of receive antennas $R$ even for $R > C$. 

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Conclusions

✓ An algorithm based on blind signal subspace estimation was proposed.

✓ Sufficient power margin is needed between desired signal and interference.

✓ Inter-cell interference is managed without coordination: only power control and power controlled hand-off are required.

✓ Low complexity detection/decoding working in the signal subspace.
✓ The algorithm works also at a very low coherence time.

✓ It benefits from an increase of $R$ also with very low coherence time.

✓ Pilot decontamination is not a fundamental property of massive MIMO systems, but appears with linear estimation.

✓ The effects of $T, C, \text{ and } R$ on performance not completely understood.
Future Work

• Massive MIMO in TDD mode:
  – Refine the estimation of the projection subspace for real systems with non vanishing ratio $\frac{T}{R}$ in TDD;
  – Robust eigenvalue/vector separation also for edge-cell terminals;
  – Study of beamforming in downlink (beamforming in the projection subspace or in the original channel);

• Massive MIMO in FDD mode:
  – Exploitation of the correlation matrix reciprocity to extend previous results;

• Distributed massive MIMO:
  – Pathloss lowers diversity gain: what should be the density of distributed antenna to maintain massive MIMO advantages or how dense should a distributed antenna system be to be a “distributed massive MIMO” system?
  – How to perform robust eigenvalue/eigenvector separation?