

Matrices and kernels with positivity structures, and the question of entrywise functions preserving them, have been studied throughout the 20th century, attracting recent interest in connection to high-dimensional covariance estimation. This is the first book to systematically develop the theoretical foundations of the entrywise calculus, focusing on entrywise operations - or transforms - of matrices and kernels with additional structure, which preserve positive semidefiniteness. Designed as an introduction for students, it presents an in-depth and comprehensive view of the subject, from early results to recent progress. Topics include: structural results about, and classifying the preservers of positive semidefiniteness and other Loewner properties (monotonicity, convexity, super-additivity); historical connections to metric geometry; classical connections to moment problems; and recent connections to combinatorics and Schur polynomials. Based on the author's course, the book is structured for use as lecture notes, including exercises for students, yet can also function as a comprehensive reference text for experts.

"The opening notes of this symphony of ideas were written by Schur in 1911. Schoenberg, Loewner, Rudin, Herz, Hiai, FitzGerald, Jain, Guillot, Rajaratnam, Belton, Putinar, and others composed new themes and variations. Now, Khare has orchestrated a masterwork that includes his own harmonies in an elegant synthesis. This is a work of impressive scholarship."

—Roger Horn, University of Utah (Retired)

"It has been known since the classical product theorem of Schur that the positivity properties of matrices can interact well with entrywise transformations. This monograph systematically explores this surprisingly rich interaction and its connections to such topics as metric geometry or the theory of Schur polynomials; it will be a useful resource for researchers in this area."

—Terence Tao, University of California, Los Angeles

यथा शिखा मयूराणां नागानां मणयो यथा ।
तथा वेदाङ्गशास्त्राणां गणितं मूर्धनि स्थितम् ॥
As are the crests on the heads of peacocks,
as are the gems on the hoods of cobras,
so is mathematics, at the top of all sciences.

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Apoorva Khare



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