

---

# MA 312 Commutative Algebra / Jan–April 2020

(BS, Int PhD, and PhD Programmes)

---

Download from : <http://www.math.iisc.ernet.in/patil/courses/Current Courses/...>

---

Tel : +91-(0)80-2293 3212/09449076304

E-mails : [patil@math.iisc.ernet.in](mailto:patil@math.iisc.ernet.in)

---

Lectures : Tuesday and Thursday ; 15:30–17:00

Venue: MA LH-5 / LH-1

---

Seminars :

---

Midterms :

---

Final Examination :

---

---

## Syllabus and References

---

---

### • • Syllabus

**1. Rings and Ideals :** Rings and ring homomorphisms, Ideals, Quotient rings, Operations on ideals, Prime and maximal ideals, Nilradical and Jacobson radical.

**2. Modules :** Modules and module homomorphisms, Submodules and quotient modules, Operations on submodules, Direct sums and Direct product, Finitely generated modules, Free modules, Exact sequences, Tensor product of modules and its properties. The functors Hom and tensor product.

**3. Algebras :** Algebras over commutative rings, Examples. Polynomial algebras and its universal property. Unique Factorization domains (UFDs).

**4. Spectrum and Zariski Topology :** The  $K$ -spectrum of an algebra over a field  $K$ , Prime Spectrum and Maximal Spectrum of a commutative ring, Algebraic Sets in Spectrums and their properties, Examples. Zariski topology on Spectrums.

**5. Finite and Finite type algebras :** Algebraic and Integral elements over commutative rings. Classical Hilbert's Nullstellensatz and its equivalent forms.

**6. Rings and Modules with Chain Conditions :** Ascending and Descending chain conditions on modules, Noetherian and Artinian Modules, Noetherian and Artinian rings, Hilbert's Basis Theorem.

**7. Rings and Modules of Fractions :** Definition and Universal property, Ideal structure in the rings of fractions, Local-Global principle.

**8. Primary Decomposition :** Primary decomposition for modules, Uniqueness of isolated primary components. Associated prime ideals, Support of a module.

**9. Integral Extensions :** Integral dependence, Lying over, Going-up and Going-down theorems. Integrally closed domains, Transcendence degree, Noether's Normalisation Lemma (NNL) and its consequences.

**10. Integrally closed Noetherian rings :** Discrete valuation rings and Dedekind domains, Fractionary ideals, Integral extensions of Noetherian domains, Galois groups and prime ideals.

---

### • • Texts/References

[1] Atiyah, M. F. and Macdonald, I. G. : *Introduction to Commutative Algebra*, Addison-Wesley, Reading, Massachusetts / London, 1969.

[2] Patil, D. P. and Storch, U. : *Introduction to Algebraic Geometry and Commutative Algebra*, IISc Lecture Notes Series, No. 1, IISc Press/World Scientific Publications Singapore / Chennai, 2010, pp. x

+ 207, ISBN-13 978-981-4304-56-6, — Indian Edition Published by Cambridge University press India Pvt. Ltd. 2012, pp. x+209, ISBN 978-981-4307-58-1. MR#2648005 (2011f:13001)

[3] **Singh, Balwant** : *Basic Commutative Algebra*, World Scientific Publications, Singapore / Chennai, 2011.

[4] **Raghavan, S., B. Singh and Sridharan, R.** : *Homological Methods in Commutative Algebra*, TIFR Mathematical Pamphlet Number 5, Oxford University Press, 1977.

[5] **Serre, J.-P.** : *Local Algebra* (Translated from French), Springer Monographs in Mathematics, Springer-Verlag, Berlin / Heidelberg, 2000.

[6] **Zariski, O. and Samuel, P.** : *Commutative Algebra*, Vols. I , II, Van Nostrand, Princeton, 1958 , 1960.

---