E0 219 Linear Algebra and Applications / August-December 2016

(ME, MSc. Ph. D. Programmes)

Download from : http://www.math.iisc.ernet.in/patil/courses/courses/Current Courses/...

Tel : +91-(0)80-2293 2239/(Maths Dept. 3212) **E-mails :** dppatil@csa.iisc.ernet.in / patil@math.iisc.ernet.in

Lectures : Monday and Wednesday ; 11:00–12:30 **Venue:** CSA, Lecture Hall (Room No. 117)

Corrections by : **Nikhil Gupta** (nikhil.gupta@csa.iisc.ernet.in ; **Lab No.: 303**) /
Vineet Nair (vineetn90@gmail.com ; **Lab No.: 303**) /
Rahul Gupta (rahul.gupta@csa.iisc.ernet.in ; **Lab No.: 224**) /
Sayantan Mukherjee (meghanamande@gmail.com ; **Lab No.: 253**) /
Palash Dey (palash@csa.iisc.ernet.in ; **Lab No.: 301 , 333 , 335**)

Midterms : 1-st Midterm : Saturday, September 17, 2016; 15:00 – 17:00 **2-nd Midterm :** Saturday, October 22, 2016; 15:00 – 17:00

Final Examination : December ??, 2016, 09:00 – -12:00

Evaluation Weightage : Assignments : 20% **Midterms (Two) :** 30% **Final Examination :** 50%

Range of Marks for Grades (Total 100 Marks)						
	Grade S	**Grade A**	**Grade B**	**Grade C**	**Grade D**	**Grade F**
Marks-Range	> 90	76 — 90	61 — 75	46 — 60	35 — 45	< 35

	Grade A$^+$	**Grade A**	**Grade B$^+$**	**Grade B**	**Grade C**	**Grade D**	**Grade F**
Marks-Range	> 90	81 — 90	71 — 80	61 — 70	51 — 60	40 — 50	< 40

2. Vector Spaces

Submit a solution of the $*$-Exercise ONLY. **Due Date :** Wednesday, 17-08-2011 (Before the Class)

Let $\mathbb{K}$ denote either the field $\mathbb{R}$ of real numbers or the field $\mathbb{C}$ of complex numbers.

2.1 Let K be a field and let I be an index set.

(a) The subsets $(K^I)_{\rm finite} := \{f \in K^I \mid f(I) \text{ is finite}\}$, $(K^I)_{\rm countable} := \{f \in K^I \mid f(I) \text{ is countable}\}$ and $(\mathbb{K}^I)_{\rm bdd} := \{f \in \mathbb{K}^I \mid f \text{ is bounded}\}$ are $\mathbb{K}$-subspaces of the K-vector space $\mathbb{K}^I$.

(b) The set $W_{\rm even}$ (resp. $W_{\rm odd}$) of all even (resp. odd) functions[1] $\mathbb{R} \to \mathbb{K}$ is a $\mathbb{K}$-subspaces of $\mathbb{K}^{\mathbb{R}}$. Further, show that $W_{\rm even} \cap W_{\rm odd} = 0$ and $W_{\rm even} + W_{\rm odd} = \mathbb{K}^{\mathbb{R}}$.

(c) The set of all functions $f : \mathbb{C} \to \mathbb{C}$ with $\lim\limits_{z\to\infty} f(z) = 0$ is a $\mathbb{C}$-subspace of the vector space $\mathbb{C}^{\mathbb{C}}$ of all $\mathbb{C}$-valued functions on $\mathbb{C}$.

2.2 Let V be a vector space over a field K with a field with $|K| \geq n$ and let $V_1, \ldots, V_n$ be K-subspaces of V. If $V_i \neq V$ for every $1 \leq i \leq n$ then show that $V_1 \cup \cdots \cup V_n \neq V$. Show by an example that the condition $|K| \geq n$ is necessary. (**Hint :** By induction on n, assume that $V_1 \cup \cdots \cup V_{n-1} \neq V$. Choose $x \in V_n$ with $x \notin V_1 \cup \cdots \cup V_{n-1}$ and $y \in V$ with $y \notin V_n$. Now consider the set $\{ax + y \mid a \in K\}$ which has at least n distinct elements.)

2.3 For subspaces U, U', W, W' of a vector space V over a field K, show that :

(a) The subset $V \smallsetminus (U \smallsetminus W)$ is a subspace of V if and only if $U = V$ or $U \subseteq W$.

(b) $U + (U' \cap W) \subseteq (U + U') \cap (U + W)$.

(c) $U \cap (U' + W) \supseteq (U \cap U') + (U \cap W)$.

(d) (M o d u l a r l a w) If $U \subseteq U'$, then $U + (U' \cap W) = U' \cap (U + W)$.

(e) If $U \cap W = U' \cap W'$, then $U = \big(U + (W \cap U')\big) \cap \big(U + (W \cap W')\big)$.

***2.4** Let K be a field and $K[X]$ be the set of polynomials with coefficients in K. Let $\varepsilon : K[X] \to K^K$ be the (evaluation) map $F(X) \mapsto (a \mapsto F(a))$. Show that

(a) ε is injective if and only if K is not finite. (**Hint :** Use the Identity Theorem for Polynomials, see Supplement S2.6 (d).)

(b) ε is surjective if and only if K is finite. (**Hint :** *Polynomial Interpolation*! Supplement S2.8.)

[1] A function $f : \mathbb{R} \to \mathbb{K}$ is called e v e n (respectively, o d d) if $f(-x) = f(x)$ (respectively, $f(-x) = -f(x)$) for all $x \in \mathbb{R}$. For example, the sine sin : $\mathbb{R} \to \mathbb{R}$ (respectively, cosine cos; $\mathbb{R} \to \mathbb{R}$) function is an odd (respectively, even) function.